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Quantum kinetic description of nonlinear collisional absorption in dense plasmas

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Abstract

The description of energy absorption in a fully ionized plasma in an electric field is generalized to arbitrary frequencies and field strengths. The limiting cases of weak and of high fields as well as the frequency dependence of the energy absorption rate are discussed.

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1. Introduction

In papers on collisional absorption in strong high-frequency fields, it is usually assumed that the contribution of collisions in the current balance equation is small compared to that of the field [1–4]. This assumption is reflected by the so-called Silin ansatz [1] or by a transformation into the Kramers–Henneberger frame [5, 6] oscillating with the quiver velocity of free particles in a field. In linear response approaches, on the other hand, generalized Drude formulae with dynamical electron–ion collision frequencies can be derived [7, 8]. The two approaches coincide in the case of weak fields and high frequencies. In this paper, a generalization for arbitrary frequencies and field strengths is given. Starting point is the electrical current balance equation which is nonlinear in the current j. The induced field follows from the mean-field term. In the collisional part, it is possible to introduce a generalized dynamical electron–ion collision frequency valid for arbitrary field strength. In the linear response regime, the familiar expressions are obtained. In the vicinity of the plasma frequency, however, there could occur resonance phenomena, i.e., the occurrence of large current amplitudes for small field amplitude E_0. Therefore, in this area, the electron–ion collision frequency depends on the current in a nonlinear way. This might have some consequences for physical properties like energy absorption or reflectivity.

2. Electrical current balance

We consider a fully ionized plasma. The balance equation for the electrical current density can be written in the following form:

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{j}(t) - \varepsilon_0\omega_p^2\mathbf{E}(t) + \omega_p^2\int_{t_0}^{t}\mathrm{d}\bar{t}\,\mathbf{j}(\bar{t}) = \sum_c\int\frac{\mathrm{d}^3p_c}{(2\pi\hbar)^3}\frac{e_c\mathbf{p}_c}{m_c}I_c(\mathbf{p}_c,t), \tag{1}$$

with the plasma frequency $\omega_p^2 = \sum_c \frac{e_c^2 n_c}{\varepsilon_0 m_c}$ and I_c being the general collision integral for species c. In contrast to former papers [9, 10], the third term on the left-hand side is included describing a polarization stemming from the mean-field contribution. The right-hand side of the above equation describes the friction $-\mathbf{R}$ due to collisions. The balance equation can be written as

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{j}(t) + \mathbf{R}\{\mathbf{j}\} + \omega_p^2\int_{t_0}^{t}\mathrm{d}\bar{t}\,\mathbf{j}(\bar{t}) = \varepsilon_0\omega_p^2\mathbf{E}(t). \tag{2}$$

Introducing the polarization $\mathbf{P}(t) = \int_{t_0}^{t}\mathrm{d}\bar{t}\,\mathbf{j}(\bar{t})$, we would get an equation for a driven harmonic oscillator with nonlinear friction. The friction term is a nonlinear non-Markovian functional of the current which reads (to lowest order in the electron–ion interaction)

$$\mathbf{R}\{\mathbf{j}\} = \frac{1}{\mathrm{i}}\int\frac{\mathrm{d}^3q}{(2\pi\hbar)^3}\mathbf{q}\int_0^{t-t_0}\mathrm{d}\tau\,F(\mathbf{q};\tau)\exp\left\{-\frac{\mathrm{i}}{\hbar}\frac{1}{n_e e_e}\mathbf{q}\cdot\int_{t-\tau}^{t}\mathrm{d}\bar{t}_1\mathbf{j}(\bar{t}_1)\right\}, \tag{3}$$

with the abbreviation

$$F(\mathbf{q},\tau) = \frac{2\pi}{\hbar}\left(\frac{e_e}{m_e} - \frac{e_i}{m_i}\right)V_{ei}^2(q)\big[\mathcal{S}_{ee}(\mathbf{q};\tau)\mathcal{L}_{ii}^A(\mathbf{q};-\tau) + \mathcal{L}_{ee}^R(\mathbf{q};\tau)\mathcal{S}_{ii}(\mathbf{q};-\tau)\big], \tag{4}$$

and the dynamic structure factors $\mathcal{S}$ and density response functions $\mathcal{L}$ of the two subsystems, electrons and ions.

There are two well-known limiting cases: in linear response, the exponential function can be expanded and the friction term is linear in the current with the dynamical collision frequency as prefactor

$$-\mathrm{i}\omega j(\omega) + \nu(\omega)j(\omega) + \frac{\omega_p^2}{-\mathrm{i}\omega}j(\omega) = \varepsilon_0\omega_p^2 E(\omega), \tag{5}$$

where $E(\omega)$ is the Fourier component of the external field. For strong fields, on the other hand, the collisions are a small correction to the quiver motion of the particles and one can use in the exponential

$$\mathbf{j} \approx \mathbf{j}^0 = \sum_a\frac{e_a^2 n_a}{m_a}\int_{t_0}^{t}\mathrm{d}t'\mathbf{E}(t'), \tag{6}$$

which corresponds to the so-called Silin ansatz. For a harmonic field $\mathbf{E} = \mathbf{E}_0\cos\omega t$, this leads to expansions [3, 4, 6] in terms of Bessel functions J_n, see also [11], and the current $\mathbf{j}(t) = \sum_{n=1}^{\infty}\mathbf{j}_{n,0}\sin(n\omega t + \phi_n)$ can be calculated as a function of the electrical field.

In the general case, equation (2) cannot be solved simply for j. Therefore, we propose the following scheme. As the higher harmonics are much smaller than the fundamental [4], in $R\{j\}$ the approximation $j(t) = j_{1,0}\sin\omega t$ is used. Then amplitude and phase of the external field $E(t) = E_0\cos(\omega t - \phi_1)$ are calculated as a function of $j_{1,0}$. After that, this relation has to be inverted. The higher harmonics of the current can also be calculated first as functions of $j_{1,0}$.

Treating $j_{1,0}$ as the independent variable, the amplitude and the phase of the external electric field follow from

$$E(\omega) = \frac{1}{\varepsilon_0 \omega_p^2} \left[\omega + \mathrm{i}\nu_1(\omega, j_{1,0}) - \frac{\omega_p^2}{\omega} \right] \frac{j_{1,0}}{2}. \tag{7}$$

With $E(\omega) = \frac{E_0}{2} \mathrm{e}^{\mathrm{i}\phi_1}$, amplitude and phase are given by

$$\begin{aligned} E_0 &= \frac{j_{1,0}}{\varepsilon_0 \omega_p^2} \sqrt{\left[\omega - \mathrm{Im}\,\nu_1(\omega, j_{1,0}) - \frac{\omega_p^2}{\omega} \right]^2 + [\mathrm{Re}\,\nu_1(\omega, j_{1,0})]^2} \\ \tan\phi_1 &= \frac{\mathrm{Re}\,\nu_1(\omega, j_{1,0})}{\omega - \mathrm{Im}\,\nu_1(\omega, j_{1,0}) - \frac{\omega_p^2}{\omega}}. \end{aligned} \tag{8}$$

Note that in the vicinity of the plasma frequency, $\omega \approx \omega_p$, there could occur resonance phenomena, i.e., the occurrence of large current amplitude $j_{1,0}$ for small field amplitude E_0. Although the external field is small, the collision frequency depends on the current in a nonlinear way, cf equation (9).

3. Collision frequency

The generalized collision frequency is given by

$$\nu_1(\omega) = \mathrm{i}\frac{2}{j_{1,0}} \sum_{m=-\infty}^{\infty} \int \frac{\mathrm{d}^3 q}{(2\pi\hbar)^3} \mathbf{q}\cdot\mathbf{n} F(\mathbf{q}; m\omega) J_{m-1}(z) J_m(z), \tag{9}$$

with $z = \mathbf{q}\cdot\mathbf{n}v_{1,0}/(\hbar\omega)$ and $\mathbf{n}v_{1,0} = \mathbf{j}_{1,0}/(n_e e_e)$ and $\mathbf{n}$ being the unit vector in field direction.

Let us have a look at the linear response case $j_{1,0} \to 0$. To lowest order one has $J_n = (z/2)^n/n!$, and only the terms with $m = 0$ and $m = 1$ contribute in the above equation. It follows that

$$\lim_{j_{1,0}\to 0} \nu_1(\omega) = \mathrm{i}\frac{1}{n_e e_e} \int \frac{\mathrm{d}^3 q}{(2\pi\hbar)^3} \frac{1}{\hbar\omega} (\mathbf{q}\cdot\mathbf{n})^2 [F(\mathbf{q}; \omega) - F(\mathbf{q}; 0)]. \tag{10}$$

For high frequencies $F(\mathbf{q}, \omega) \approx e_e/(m_e\hbar) V_{ei}^2(q) n_i \mathcal{S}_{ii}(\mathbf{q}) \mathcal{L}_{ee}^R(\mathbf{q}; \omega)$ and introducing the dielectric function of the electron subsystem, we have

$$\lim_{j_{1,0}\to 0} \nu_1(\omega) = \mathrm{i}\frac{n_i e_i^2}{n_e m_e \hbar^2} \int \frac{\mathrm{d}^3 q}{(2\pi\hbar)^3} \frac{1}{\omega} (\mathbf{q}\cdot\mathbf{n})^2 \mathcal{S}_{ii}(q) V(q) \left[\varepsilon_{ee}^{-1}(\mathbf{q}; \omega) - \varepsilon_{ee}^{-1}(\mathbf{q}; 0)\right]. \tag{11}$$

With the dielectric function in random phase approximation, this is a well-known expression [7, 8].

4. Higher harmonics

From equation (2) there follows for the higher harmonics of the current $l > 1$

$$\mathbf{j}_l(\omega) = -\frac{1}{\left(l\omega - \frac{\omega_p^2}{l\omega}\right)} \sum_{m=-\infty}^{\infty} \frac{1}{\mathrm{i}^l} \int \frac{\mathrm{d}^3 q}{(2\pi\hbar)^3} \mathbf{q} F(\mathbf{q}; m\omega) J_{m-l}(z) J_m(z). \tag{12}$$

In the following, we assume that the function F depends on the modulus of $\mathbf{q}$ only. For the Bessel functions there holds that $J_{m-l}(-z)J_m(-z) = (-1)^l J_{m-l}(z)J_m(z)$. Therefore, the integrand in equation (12) is an odd function of $\mathbf{q}$ for even l and an even function for odd l and consequently only odd harmonics of the current can exist.

5. Energy absorption

The energy absoption rate is given by

$$\nu_E = \frac{\langle \mathbf{j} \cdot \mathbf{E} \rangle}{\langle \varepsilon_0 \mathbf{E} \cdot \mathbf{E} \rangle} = \frac{\mathbf{j}_{1,0} \cdot \mathrm{Im}\, \mathbf{E}(\omega)}{\varepsilon_0 2[(\mathrm{Re}\, \mathbf{E}(\omega))^2 + (\mathrm{Im}\, \mathbf{E}(\omega))^2]}$$
$$= \frac{\omega_p^2}{\left[\omega - \frac{\omega_p^2}{\omega} - \mathrm{Im}\, \nu_1(\omega)\right]^2 + [\mathrm{Re}\, \nu_1(\omega)]^2} \mathrm{Re}\, \nu_1(\omega). \qquad (13)$$

Here, the brackets denote cycle-averaged quantities [4, 12]. In the second line of the equation, the dependence on the electrical field strength is implicit via the current amplitude $j_{1,0}$ in the generalized dynamical collision frequency. Near the plasma frequency, the energy absorption is enhanced due to the collective motion of the electrons. In the high-frequency limit, however, we get the familiar expression $\nu_E = \omega_p^2/\omega^2\, \mathrm{Re}\, \nu_1(\omega)$. Calculations as well as details of the derivation will be given elsewhere.

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References

[1] Silin V P 1964 *Zh. Eksp. Teor. Fiz.* **47** 2254
Silin V P 1965 *Sov. Phys.—JETP* **20** 1510 (Engl. Transl.)
[2] Klimontovich Yu L 1975 *Kinetic Theory of Nonideal Gases and Nonideal Plasmas* (Moscow: Nauka) (in Russion) (Engl. Transl.: Pergamon, Oxford, 1982)
[3] Kremp D, Bornath Th, Bonitz M and Schlanges M 1999 *Phys. Rev.* E **60** 4725
[4] Bornath Th, Schlanges M, Hilse P and Kremp D 2001 *Phys. Rev.* E **64** 26414
[5] Decker C D, Mori W B, Dawson J M and Katsouleas T 1994 *Phys. Plasmas* **1** 4043
[6] Kull H-J and Plagne L 2001 *Phys. Plasma* **8** 5244
[7] Bekefi G 1966 *Radiation Properties in Plasmas* (New York: Wiley)
[8] Reinholz H, Redmer R, Röpke G and Wierling A 2000 *Phys. Rev.* E **62** 5648
[9] Bornath Th, Schlanges M, Hilse P and Kremp D 2003 *J. Phys. A: Math. Gen.* **36** 5941
[10] Kremp D, Bornath Th, Bonitz M, Semkat D and Schlanges M 2005 *Contrib. Plasma Phys.* **45** 396
[11] Schlanges M, Bornath Th and Hilse P 2006 *J. Phys. A: Math. Gen.* **39** 4703
[12] Bornath Th, Schlanges M, Hilse P and Kremp D 2004 *Nonequilibrium Physics at Short Time Scales* ed K Morawetz (Berlin: Springer) p 154